Integrals

The function F(x) is called anti-derivative or integral or primitive of the function F(x) and C is called constant of integration or arbitrary constant.

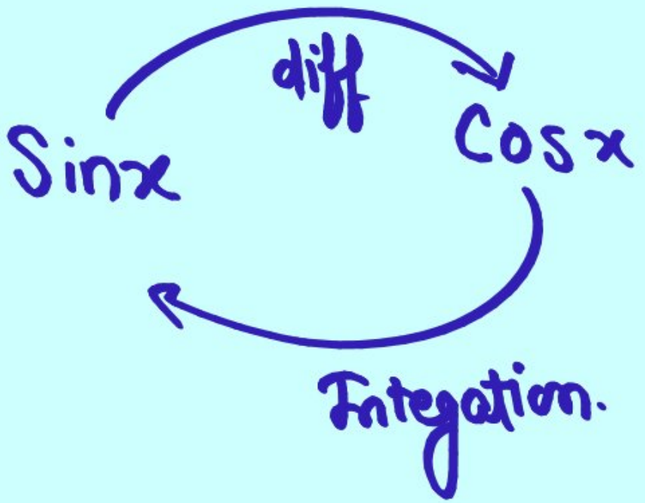
$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\int \sin x\, dx = -\cos x + c$$

$$\int \cos x\, dx = \sin x + c$$



Fundamental Integration Formulas

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \operatorname{cosec}^2 x \, dx = -\cot x + c$$

$$\int \sec x \tan x \, dx = \sec x + c$$

$$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \log x = \frac{1}{x}$$

$$\frac{d}{dx} \frac{a^x}{\log a} = a^x$$

$$\int e^x dx = e^x + c$$

$$\int \frac{1}{x} dx = \log|x| + c$$

$$\int a^x dx = \frac{a^x}{\log a} + c$$

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Evaluate the Following

1. i) $\int x^5 \, dx$

$$I = \int x^5 \, dx$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

$$n = 5$$

$$I = \frac{x^{5+1}}{5+1} + c$$

$$I = \frac{x^6}{6} + c$$

ii) $\int \frac{1}{x^3}\,dx$

$$I = \int \frac{1}{x^3}\,dx$$

$$I = \int x^{-3}\,dx$$

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + c}$$

$$I = \frac{x^{-3+1}}{-3+1} + c$$

$$I = \frac{x^{-2}}{-2} + c$$

$$\boxed{I = -\frac{1}{2x^2} + c}$$

$iii) \int \frac{1}{\sqrt{y}} dx$

2.i) $\int \frac{1}{\sqrt[3]{x}} dx$

ii) $\int sin^2 x cosec^2 x dx$

$$I = \int \sin^2 x \, \operatorname{Cosec}^2 x \, dx$$

$$= \int \sin^2 x \times \frac{1}{\sin^2 x} \, dx$$

$$I = \int 1 \cdot dx$$

$$I = x + c$$ Ans

$$\operatorname{Cosec} x = \frac{1}{\sin x}$$

$$\frac{d}{dx} 1 = 0$$

$$\int 1 \cdot dx = x + c$$

iii) $\int \sin x \sec^2 x dx$

but

$$I = \int \sin x \sec^2 x \, dx$$

$$= \int \sin x \times \frac{1}{\cos^2 x} dx$$

$$= \int \frac{\sin x}{\cos x} \times \frac{1}{\cos x} dx$$

$$= \int \tan x \sec x \, dx$$

$$\boxed{\int \sec x \tan x \, dx = \sec x + c}$$

$$\boxed{I = \sec x + c}$$

3. i) $\int 5^x dx$

ii) $\int 7^x e^x dx$

$$I = \int 7^x e^x \, dx$$

$$= \int (7e)^x \, dx$$

$$\int a^x \, dx = \frac{a^x}{\log a} + c$$

$$I = \frac{(7e)^x}{\log 7e} + c$$

iii) $\int \frac{4^x}{9^x} dx$

4 i) $\int \frac{\sin x}{1-\sin^2 x}\, dx$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$$I = \int \frac{\sin x}{1-\sin^2 x}\, dx$$

$$= \int \frac{\sin x}{\cos^2 x}\, dx$$

$$= \int \frac{\sin x}{\cos x} \times \frac{1}{\cos x}\, dx$$

$$I = \int \tan x \sec x\, dx$$

$$\boxed{I = \sec x + c}$$

ii) $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\, dx$

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$$I = \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\, dx$$

$$= \int \frac{1 - 2\sin^2 x + 2\sin^2 x}{\cos^2 x}\, dx$$

$$= \int \frac{1}{\cos^2 x}\, dx$$

$$= \int \sec^2 x\, dx$$

$$\boxed{I = \tan x + c}$$

iii) $\int \sqrt{\frac{1}{2}(1+\cos 2x)}\,dx$

$$I=\int \sqrt{\frac{1}{2}(1+2\cos^2 x-1)}\,dx$$

$$=\int \sqrt{\frac{1}{2}(2\cos^2 x)}\,dx$$

$$=\int \sqrt{\cos^2 x}\,dx$$

$$\boxed{I=\int \cos x\,dx=-\sin x+c}$$

5 i) $\int e^{2\log x} x^{-3} dx$

$$I = \int e^{2\log x} x^{-3} dx$$

$$= \int e^{\log x^2} x^{-3} dx$$

$$= \int x^2 e^{-3} dx$$

$$= \int \frac{x^2}{x^3} dx$$

$$I = \int \frac{1}{x} dx = \log x + c$$

$$n \log m = \log m^n$$

ii) $\int 5^{2\log_5 x} dx$

iii) $\int 2^{\log_4 x}\,dx$

$$I = \int 2^{\log_4 x}\,dx$$

$$= \int 2^{\log_{2^2} x}\,dx$$

$$= \int 2^{\frac{1}{2}\log_2 x}\,dx$$

$$= \int \textcircled{2}^{\log_{\textcircled{2}} x^{1/2}}\,dx$$

$$= \int x^{1/2}\,dx$$

$$\int x^{1/2}\,dx$$

$$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{x^{3/2}}{3/2} + c$$

$$e^{\log_{\textcircled{e}} x} = x$$